Displacement, velocity and acceleration as a function of time

We have spent a lot of time analysing the position, velocity and acceleration of various objects. Usually we have model the relations as having constant acceleration.

Definition. The **velocity** of a particle with displacement $x(t)$ is

$$v(t) = \frac{d}{dt}(x(t)) = x'(t) = \dot{x}(t)$$

The **acceleration** is:

$$\begin{aligned} a(t) &= \frac{d}{dt}(v(t)) = v'(t) = \dot{v}(t) \\ &= \frac{d^2}{dt^2}(x(t)) = x''(t) = \ddot{x}(t) \end{aligned}$$

Fact — We can use the chain rule to show that:

$$a(t) = \frac{d}{dt}(v(t)) = \frac{dv}{dx}\frac{dx}{dt} =$$

Example
If $v = 3x^2 - 4x$, find (a) v when $x = 2\,\text{m}$ and (b) a when $x = 2\,\text{m}$

Example (Separating Variables)
If $v = \frac{20}{3x-2}$, find (a) v when $x = 4\,\text{m}$, (b) x when $v = 5\,\text{ms}^{-1}$, (c) t when $x = 20\,\text{m}$, given that $x = 0$ when $t = 0$

Example

If $a = 3x + 5$ and initially $v = 1\,\text{ms}^{-1}$ when $x = 1\,\text{m}$, find v when $x = 2\,\text{m}$

Example

If $a = x + 5$ and initially $x = 0\,\text{m}$ and $v = 5\,\text{ms}^{-1}$, find (a) and expression for v in terms of x, (b) and expression fo t in terms of x

Example

If $a = \frac{4}{v^3}$, find t when $v = 2\,\text{ms}^{-1}$ given that when $t = 0, v = 0$.

Example

If $a = 4 + 3v$, find x when $v = 2\,\text{ms}^{-1}$ given that when $x = 0, v = 0$.

Variable Force

We are well acquainted with $F = ma$ by now, but we can also solve problems which account for F and a varying with time, for example we could solve equations of the form $F(t) = m\frac{dv}{dt}$ or $F(x) = mv\frac{dv}{dx}$.

Example

If $F = 3t + 1, m = 4\,\text{kg}$ and the body is initially at rest at point O, find (a) v when $t = 2\,\text{s}$, (b) x when $t = 2\,\text{s}$

Example

If $F = 5x + 6, m = 4\,\text{kg}$ and the body is initially at rest at point O, find (a) v when $x = 4\,\text{m}$, (b) x when $v = 9\,\text{ms}^{-1}$

Example

A body of mass 5 kg falls under gravity and reaches a terminal velocity V ms^{-1} downwards. Given the body experience a resisting force of $(0.04v^2)$N where v is the speed of the body in ms^{-1}, determine V and find the time taken for the body to reach a speed of $\frac{4V}{5}$ ms^{-1}

Energy and Power

Fact —

$$\text{Work Done} = \int \text{Force}\, dx$$

Example

After release from rest at point O, a body of mass 1 kg falls under gravity against a resistance $\frac{24}{25}s$ N, where s metres is the distance the body is below O at any instant. Find the amount of work done by the body against the resistance, from release until it passes through a point P, 10 m below O, and find the speed of the body at that instant.